3

BASIC EQUATION OF MOTION

3.1 FORMULATION IN TOTAL DISPLACEMENTS

3.1.1 Flexible Base

To derive the basic equations of motion, it is sufficient to examine a single structure embedded in soil for earthquake excitation (Fig. 3-1). The structure's base consisting of the basemat and of the adjacent walls is assumed at first to be flexible. The structure is discretized schematically as shown. Subscripts are used to denote the nodes of the discretized system. The nodes located on the structure–soil interface are denoted by b (for base), the remaining nodes of the structure by s. In the substructure method no nodes are introduced in the interior of the soil.

The dynamic system consists of two substructures, the actual structure and the soil with excavation. To differentiate between the various subsystems, superscripts are used when necessary. The structure is indicated by s (when used with a property matrix), the other substructure, the soil with excavation, by g (for ground). In the following it is appropriate to work also with other subsystems for the soil (Fig. 3-2). The soil without excavation, the so-called free field, is denoted by f, and e is used to designate the excavated soil.

The dynamic equations of the motion are formulated in the frequency domain. The amplitudes of the total displacements are denoted by $\{u^t\}$, which is a function of the discrete value of the frequency ω. The subscript used to indicate a discrete value of ω in Eqs. 2.20 and 2.21 is deleted. The word "total" (superscript t) expresses that the motion is referred to an origin that does not

مدرس: دکتر علیرضا امامی (هیئت علمی دانشگاه آزاد-واحد اصفهان)

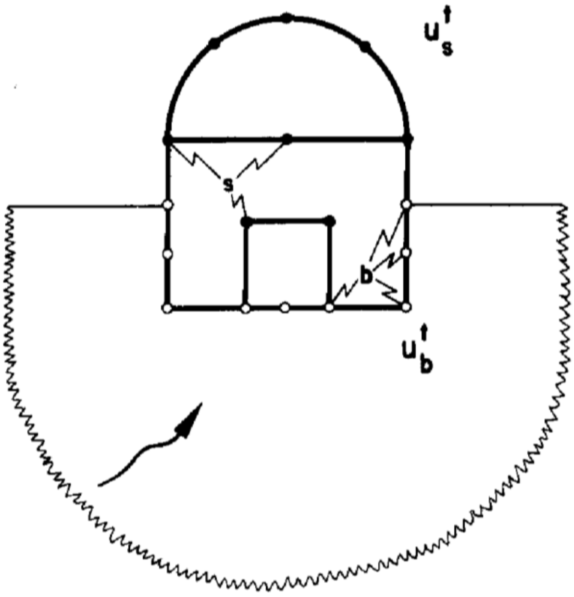


Figure 3-1 Structure–soil system.

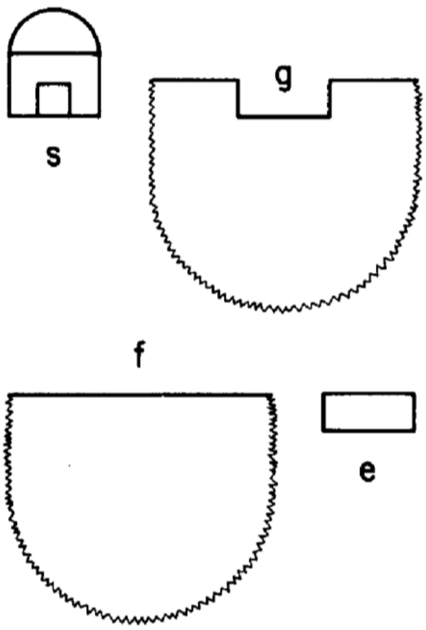


Figure 3-2 Reference subsystems.

معادلات اندرکنش خاک وسازه درحوزه‌ی فرکانس:

b: base : درجات آزادی در مرز تماس خاک وسازه. (SSI Interface) } زیر اندیس ها

s: structure : سایر درجات آزادی باقی مانده روی سازه.

s: . : سازه که روی محیط خاکبرداری شده قرار میگیرد

g: ground : محیط خاک با درنظر گرفتن عملیات خاکبرداری } بالا اندیس ها

f: free field : محیط خاک بدون عملیات خاکبرداری } بالا اندیس ها

e: excavated s. : قسمت خاکبرداری شده

$\{u^t(\omega)\}$: تغییرمکانهای کل در یک مدل گسسته به مقدار ω نسبت به یک نقطه ثابت و بدون حرکت $\{u^t\} = \{u^t(\omega)\}$

ماتریس سختی سازه: $[S] = [k](1+2\zeta i) - \omega^2[M]$ $\quad \{u^t\} = \begin{Bmatrix} \{u_s^t\} \\ \{u_b^t\} \end{Bmatrix}$

move. The order of this vector equals the number of dynamic degrees of freedom of the total discretized system. The vector $\{u^t\}$ can be decomposed into the subvectors $\{u_s^t\}$ and $\{u_b^t\}$. The vector symbols are deleted in Fig. 3-1 for the sake of conciseness. The dynamic-stiffness matrix $[S]$ of the structure, which is a bounded system, is calculated as (Eq. 2.15)

$$[S] = [K](1 + 2\zeta i) - \omega^2[M] \tag{3.1}$$

where $[K]$ and $[M]$ are the static-stiffness and mass matrices, respectively. The hysteretic-damping ratio ζ, which is independent of frequency, is assumed to be constant throughout the structure. The formulation can straightforwardly be expanded to the case of nonuniform damping. Some formulas, however, cannot be written so concisely. $[S]$ can also be decomposed into the submatrices $[S_{ss}]$,

2/ $[S] = [k](1+2\zeta i) - \omega^2 [m]$

$m\ddot{u} + c\dot{u} + ku = P(t) = P_0 e^{i\omega t} \quad ; \quad u = u_0 e^{i\omega t}$

$\underbrace{[-m\omega^2 + ic\omega + k]}_{S} u_0 = P_0$

سختی دینامیکی

$S = k - m\omega^2 + i\omega c \Rightarrow S = k + i\omega c - m\omega^2$

$S = k\left(1 + \frac{i\omega c}{k}\right) - m\omega^2$

$\frac{\omega c}{k} = \omega \frac{c}{\sqrt{k}\sqrt{k}} \times \frac{\sqrt{m}}{\sqrt{m}} = \omega \underbrace{\left\{\frac{c}{2\sqrt{km}}\right\}}_{\frac{c}{c_{cr}} = \zeta} \times 2 \underbrace{\left\{\frac{\sqrt{m}}{\sqrt{k}}\right\}}_{\omega_n} =$

$\frac{\omega c}{k} = 2\omega\zeta \times \frac{1}{\omega_n}$

$\omega = \omega_n \quad \frac{\omega c}{k} = 2\zeta \frac{\omega}{\omega_n} = 2\zeta$

$S = k\left(1 + i\frac{\omega c}{k}\right) - \cancel{m\omega^2} k(1 + 2\zeta i) - m\omega^2$

در فرم ماتریسی $[S] = [k](1+2\zeta i) - \omega^2 [m]$

$[S_{sb}]$, and $[S^s_{bb}]$. To avoid using unnecessary symbols, the superscript s (for structure) is used only when confusion would otherwise arise. The equations of motion of the structure are formulated as

$$\begin{bmatrix} [S_{ss}] & [S_{sb}] \\ [S_{bs}] & [S^s_{bb}] \end{bmatrix} \begin{Bmatrix} \{u^t_s\} \\ \{u^t_b\} \end{Bmatrix} = \begin{Bmatrix} \{P_s\} \\ \{P_b\} \end{Bmatrix} \tag{3.2}$$

where $\{P_s\}$ and $\{P_b\}$ denote the amplitudes of the loads and of the interaction forces with the soil, respectively.

The dynamic-stiffness matrix of the soil $[S^g_{bb}]$ (Fig. 3-3) is not as easy to determine as that of the structure, as the soil is an unbounded domain. Conceptionally, $[S^g_{bb}]$ could be determined by eliminating all degrees of freedom not lying on the structure–soil interface of a mesh of the soil extending to infinity. The vector $\{u^g_b\}$ denotes the displacement amplitudes of the soil with excavation for the earthquake excitation. For the reference system of the free field, $[S^f_{bb}]$ and $\{u^f_b\}$ are the dynamic-stiffness matrix and the vector of displacement amplitudes, respectively. The dynamic-stiffness matrix $[S^e_{bb}]$ of the excavated soil, a bounded domain, follows analogously from Eq. 3.1, using the properties of the soil.

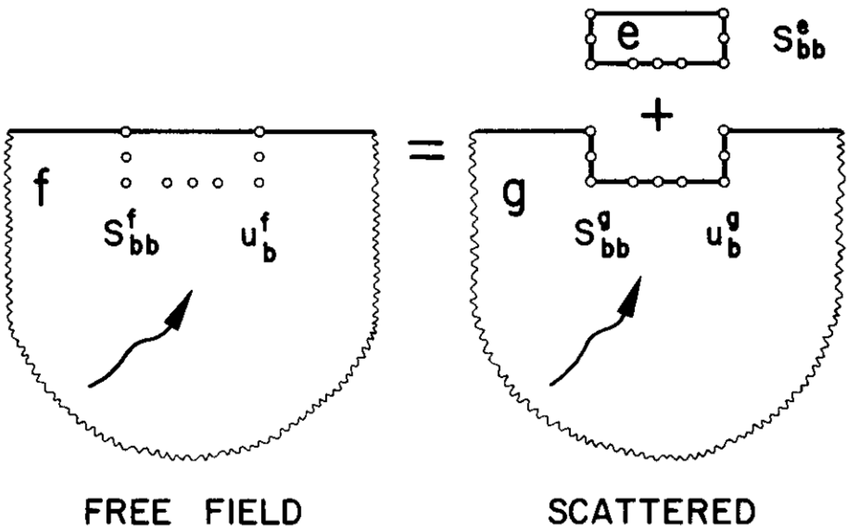
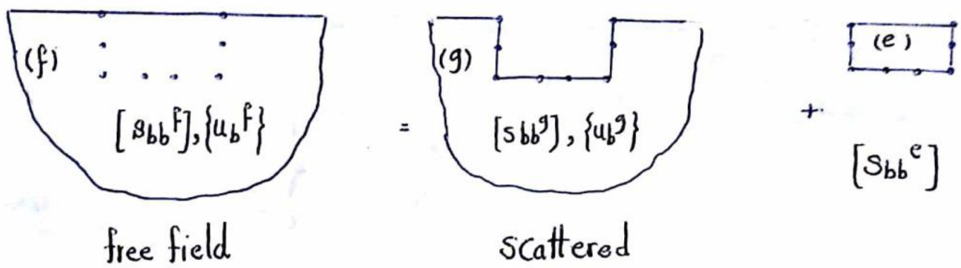


Figure 3-3 Dynamic-stiffness matrix and earthquake excitation referred to different reference systems of soil.

3/ $$\begin{bmatrix}[S_{ss}] & [S_{sb}]\\ [S_{bs}] & [S_{bb}^{s}]\end{bmatrix}\begin{Bmatrix}\{u_s^t\}\\ \{u_b^t\}\end{Bmatrix} = \begin{Bmatrix}\{P_s\}\\ \{P_b\}\end{Bmatrix}$$

باید دقت داشت که ζ نسبت میرایی هیسترزیس است که به صورت مستقل از فرکانس تحریک (ω) لحاظ شده و به طور یکنواخت برای کل سازه به کار رفته است.



$[S_{bb}^{gg}]$: ماتریس سختی خاک با انجام عملیات خاکبرداری است که درجات آزادی خارج از مرز تماس آن حذف شده است.

$\{u_b^{g}\}$: دامنه تغییر مکان محیط خاکبرداری شده در هنگام تحریک لرزه ای.

For earthquake excitation, the nodes not in contact with the soil (the subscript s stands for the structure) are not loaded. Setting $\{P_s\} = \{0\}$ in Eq. 3.2 leads to

$$[[S_{ss}][S_{sb}]]\begin{Bmatrix}\{u_s^t\}\\ \{u_b^t\}\end{Bmatrix} = \{0\} \tag{3.3}$$

Both substructures contribute to the dynamic equilibrium equations of the nodes b lying on the structure–soil interface. The contribution of the soil is discussed first. For the displacement amplitudes $\{u_b^g\}$, the interaction forces acting in the nodes b, arising from the soil with excavation, vanish, as for this loading state, the line that will subsequently form the structure–soil interface is a free surface (Figs. 3-2 and 3-3). The interaction forces of the soil will thus depend on the motion relative to $\{u_b^g\}$. They are equal to $[S_{bb}^g](\{u_b^t\} - \{u_b^g\})$. Including the contribution of the structure ($\{P_b\}$ in Eq. 3.2), the equations of motion for the nodes in contact with the soil (subscript b) are formulated as

$$[S_{bs}]\{u_s^t\} + [S_{bb}^s]\{u_b^t\} + [S_{bb}^g](\{u_b^t\} - \{u_b^g\}) = \{0\} \tag{3.4}$$

$[S_{bb}^{g}]$: ماتریس سختی خاک تا مرز بی‌نهایت و حذف درجات آزادی حاصل می‌شود که در مرز تماس با سازه نیستند.

درصورت تحریک لرزه‌ای $\{P_s\}=\{0\}$ در سازه:

$$[S_{ss}]\{u_s^t\}+[S_{sb}]\{u_b^t\}=\{0\}$$

$$\{P_b\}=-[S_{bb}^g](\{u_b^t\}-\{u_b^g\})$$

علامت منفی به خاطر کنش و واکنش است.

(۱) پی انعطاف‌پذیر:

Scattered motion: $\{u_b^g\}$

$$\rightarrow [S_{bs}]\{u_s^t\}+[S_{bb}^s]\{u_b^t\}=-[S_{bb}^g]\{u_b^t\}+[S_{bb}^g]\{u_b^g\}$$

$$\rightarrow \begin{bmatrix} [S_{ss}] & \vdots & [S_{sb}] \\ [S_{bs}] & \vdots & [S_{bb}^s]+[S_{bb}^g] \end{bmatrix}\begin{Bmatrix}\{u_s^t\}\\ \{u_b^t\}\end{Bmatrix}=\begin{Bmatrix}\{0\}\\ [S_{bb}^g]\{u_b^g\}\end{Bmatrix}$$

تعریف در محل مرز تماس خاک و سازه برای محیط خاک‌برداری شده که به آسانی

به راحتی نمی‌...

Including the contribution of the structure ($\{P_b\}$ in Eq. 3.2), the equations of motion for the nodes in contact with the soil (subscript b) are formulated as

$$[S_{bs}]\{u_s^t\}+[S_{bb}^s]\{u_b^t\}+[S_{bb}^g](\{u_b^t\}-\{u_b^g\})=\{0\} \qquad (3.4)$$

Combining Eqs. 3.3 and 3.4, the equations of motion of the total structure-soil system are

$$\begin{bmatrix} [S_{ss}] & \vdots & [S_{sb}] \\ [S_{bs}] & \vdots & [S_{bb}^s]+[S_{bb}^g] \end{bmatrix}\begin{Bmatrix}\{u_s^t\}\\ \{u_b^t\}\end{Bmatrix}=\begin{Bmatrix}\{0\}\\ [S_{bb}^g]\{u_b^g\}\end{Bmatrix} \qquad (3.5)$$

In this formulation, the earthquake excitation is characterized by $\{u_b^g\}$, that is, the motion in the nodes (which will subsequently lie on the structure–soil interface) of the ground with the excavation. This so-called "scattered" motion is not easy to determine. It is desirable to replace $\{u_b^g\}$ by $\{u_b^f\}$, the free-field motion, which does not depend on the excavation (with the exception of the location of the nodes in which it is to be calculated).

The free-field system results when the excavated part of the soil is added to the soil with excavation (Fig. 3-3). This also holds for the assembly process of the dynamic-stiffness matrices

$$[S_{bb}^e]+[S_{bb}^g]=[S_{bb}^f] \qquad (3.6)$$

$$[S^e_{bb}] + [S^g_{bb}] = [S^f_{bb}] \tag{3.6}$$

By stipulating that the "structure" consist of the excavated part of the soil only, Eq. 3.4 can be formulated for this special case. With $[S_{bs}] = [0]$, $[S^s_{bb}] = [S^e_{bb}]$, and $\{u^t_b\} = \{u^f_b\}$,

$$([S^e_{bb}] + [S^g_{bb}])\{u^f_b\} = [S^g_{bb}]\{u^g_b\} \tag{3.7}$$

results. Introducing Eq. 3.6 in Eq. 3.7 leads to

$$[S^f_{bb}]\{u^f_b\} = [S^g_{bb}]\{u^g_b\} \tag{3.8}$$

به راحتی میتوان نوشت.

$$[S_{bb}^f] = [S_{bb}^g] + [S_{bb}^e] \longrightarrow \text{با اسمبل ماتریس سختی}$$

با فرض اینکه سازه تنها ناحیه خاکبرداری شده را شامل شود. تمامی درجات آزادی از نوع b خواهند بود.

می توان نوشت:

$$[S_{bs}]\{u_s^t\} + ([S_{bb}^s] + [S_{bb}^g])\{u_b^t\} = [S_{bb}^g]\{u_b^g\}$$

$$[S_{bs}] = [0] \;;\; [S_{bb}^s] = [S_{bb}^e] \;;\; \{u_b^t\} = \{u_b^f\}$$

$$\Rightarrow [S_{bb}^f]\{u_b^f\} = [S_{bb}^g]\{u_b^g\}$$

$$\Rightarrow \begin{bmatrix} [S_{ss}] & [S_{sb}] \\ [S_{bs}] & [S_{bb}^s]+[S_{bb}^g] \end{bmatrix} \begin{Bmatrix} \{u_s^t\} \\ \{u_b^t\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [S_{bb}^f][u_b^f] \end{Bmatrix}$$

به جای $\{u_b^g\}$ بهتر است $\{u_b^f\}$ استفاده شود که به ناحیه خاکبرداری شده وابسته نیست (مگر بواسطه مکان گره‌های b که موقعیت آنها کاملا وابسته به خاکبرداری است)

This equality of forces is quite a remarkable result in its own right. Although for the substructure of the soil with excavation, the line with the nodes b (where the motion is equal to $\{u_b^g\}$) is a free surface, as discussed above, the forces $[S_{bb}^g]\{u_b^g\}$ are not zero. The influence of an exterior boundary with an applied earthquake motion also has to be taken into account when calculating the forces in nodes b. This is explained further in Section 8.2, where the basic equations of motion are rederived starting from those of the direct formulation of soil–structure interaction.

Substituting Eq. 3.8 in Eq. 3.5 results in the discretized equations of motion

$$\begin{bmatrix} [S_{ss}] & [S_{sb}] \\ [S_{bs}] & [S_{bb}^s] + [S_{bb}^g] \end{bmatrix} \begin{Bmatrix} \{u_s^t\} \\ \{u_b^t\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [S_{bb}^f]\{u_b^f\} \end{Bmatrix} \tag{3.9}$$

In this formulation in total displacements, the load vector is expressed as the product of the dynamic-stiffness matrix of the free field $[S_{bb}^f]$ (discretized in the nodes at which the structure is subsequently inserted) and of the free-field motion $\{u_b^f\}$ in the same nodes. This physical interpretation is remarkable. The load acts only on the nodes b at the base of the structure. The interaction part of soil–structure analysis is thus formulated as a so-called source problem, with the source being located on the structure–soil interface. Only outgoing waves that propagate toward infinity arise (see Section 7.1). As expected for seismic excitation, the nodes s not connected with the soil are unloaded. It should be emphasized that in the system f for which $[S_{bb}^f]$ is calculated, the soil is not excavated (Fig. 3-3). As the boundary of the free-field region is regular, $[S_{bb}^f]$ should be easier than $[S_{bb}^g]$ to calculate (see Section 7.5.3). The vector $\{u_b^f\}$ has only to be calculated in those nodes b which subsequently will lie on the structure–soil interface. Procedures to determine $[S_{bb}^g]$, $[S_{bb}^f]$ and $\{u_b^f\}$ are discussed in depth in Chapters 7 and 6, respectively. The derivation of the basic equations of motion is based on substructuring with replacement. By adding the excavated part of the soil (system e) to the irregular system g, a regular system f is formed on which the load vector depends. See Problems 3.2, 3.3, and 3.4 for further applications of this concept, which is also valid for applied loads and not only prescribed motions.

The amount of interaction of the embedded part described by the left-hand side of Eq. 3.9 depends on $[S^s_{bb}] + [S^g_{bb}]$, which, after making use of Eq. 3.6, is equal to

$$[S^s_{bb}] + [S^g_{bb}] = [S^s_{bb}] - [S^e_{bb}] + [S^f_{bb}] \tag{3.10}$$

The difference of the property matrices of the structure and of the soil in the embedded region $[S^s_{bb}] - [S^e_{bb}]$ is thus of importance. Using Eq. 3.10, Eq. 3.9 is reformulated as

$$\begin{bmatrix} [S_{ss}] & [S_{sb}] \\ [S_{bs}] & [S^s_{bb}] - [S^e_{bb}] + [S^f_{bb}] \end{bmatrix} \begin{Bmatrix} \{u^t_s\} \\ \{u^t_b\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [S^f_{bb}]\{u^f_b\} \end{Bmatrix} \tag{3.11}$$

This represents the equations of motion of a discretized system (consisting of the structure and in the embedded region of the difference of the structure and of the soil) supported on a generalized spring described by $[S^f_{bb}]$. The excitation consists of a prescribed support motion $\{u^f_b\}$ acting at the end of the generalized spring that is not connected to the structure. This is shown schematically in Fig. 3-4, where the matrix and vector symbols as well as the subscripts have been omitted. It is important to note that the support motion to be applied at the end of the generalized spring is the free-field response $\{u^f_b\}$ at the structure–soil interface and not at some (fictitious) boundary at a depth where the underlying medium can be regarded as very stiff. For instance, consider a structure supported on the surface of a layer of soil resting on rigid bedrock. The generalized spring represents the stiffness and damping of the layer built in at its base at the top of the bedrock. The applied support motion is the free-field response at the free surface of the layer and not at its base. For a better understanding of the foregoing, it is helpful to think of the generalized spring as having a length approaching zero.

با فرض اینکه سازه برابر با همان بخش خاکبرداری شده خاک لحاظ شود، آنگاه تمامی درجات آزادی متناظر با S حذف می شوند و درجات آزادی فقط از نوع $\{u_b^t\}$ خواهند بود.

$$[S_{bs}] = [0]$$

$$\{u_s^t\} = \{0\}$$

$$[S_{bb}^s] = [S_{bb}^e]$$

$$\underbrace{[S_{bs}]\{u_s^t\}}_{\{0\}} + (\underbrace{[S_{bb}^s]}_{[S_{bb}^e]} + [S_{bb}^g])\{u_b^t\} = [S_{bb}^g]\overbrace{\{u_b^g\}}^{\{u_b^f\}}$$

$$[S_{bb}^f]\{u_b^f\} = [S_{bb}^g]\{u_b^g\}$$

محاسبه $[S_{bb}^f]$ آسانتر از $[S_{bb}^g]$ است. چون نامنظمی در مرزهای آن کمتر است.

نیرو فقط در مرز تماس سازه و خاک اعمال می شود (درنتیجه پایین)

$$\rightarrow \begin{bmatrix} [S_{ss}] & [S_{sb}] \\ [S_{bs}] & [S_{bb}^s]-[S_{bb}^e]+[S_{bb}^f] \end{bmatrix} \begin{Bmatrix} \{u_s^t\} \\ \{u_b^t\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [S_{bb}^f]\{u_b^f\} \end{Bmatrix}$$

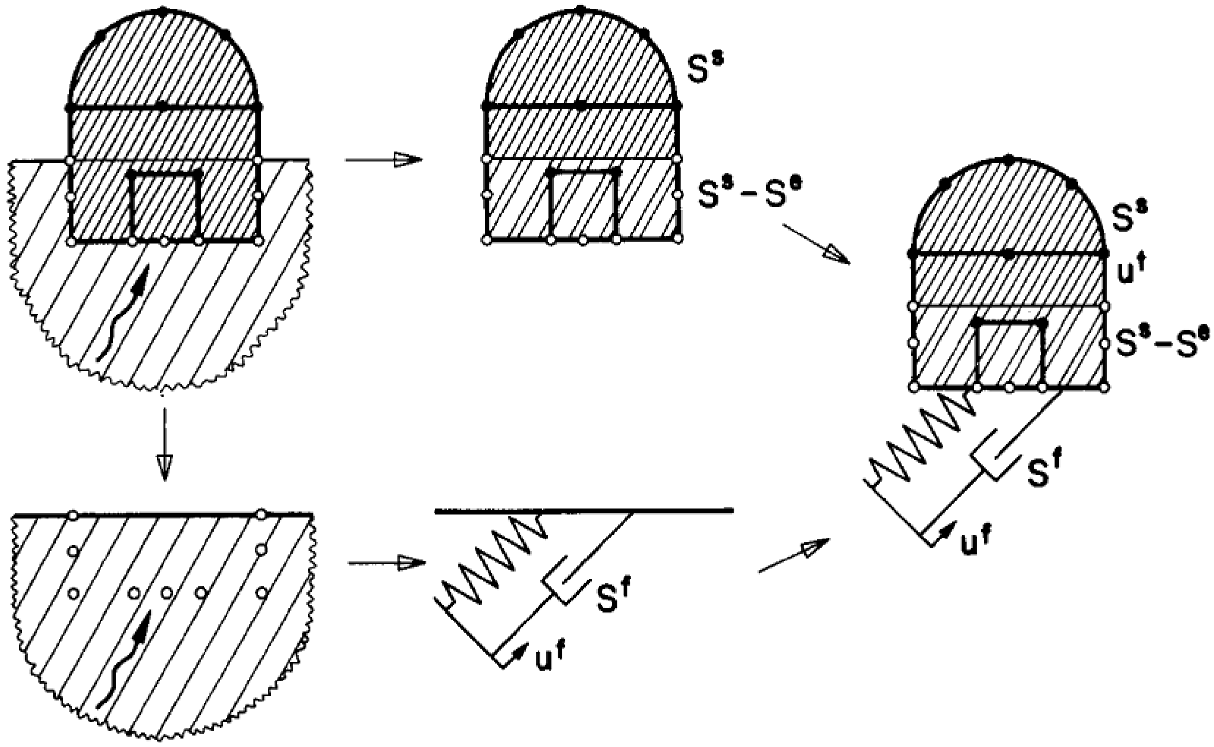


Figure 3-4 Physical interpretation of basic equation of motion in total displacements (flexible base).

Equation 3.9 describes the basic equations of motion, formulated for a structure with a flexible base. It represents a simple, but general procedure to calculate even the most general case of soil–structure interaction. All other formulations which are derived in Sections 3.2, 8.2, and 8.3 are not more powerful. They are discussed only because valuable physical insight can be gained from the equations, which work with relative displacements.

Equation 3.8 could be used to calculate the "scattered" earthquake excitation $\{u_b^g\}$:

$$\{u_b^g\} = [S_{bb}^g]^{-1}[S_{bb}^f]\{u_b^f\} \tag{3.12}$$

It is, however, unnecessary to determine this seismic motion of the soil modified by the excavation $\{u_b^g\}$ (Fig. 3-3), as Eq. 3.9 can be used. It is worth mentioning that $\{u_b^g\}$ has no real existence (i.e., it does not occur in the real soil–structure system).

3.1.2 Rigid Base

The base, consisting of the basemat and the adjacent walls, can be assumed to be rigid for many practical applications (see Section 4.3). This compatibility constraint on the structure–soil interface leads to a slight modification of the formulation. At the same time the physical significance of the terms appearing in the equations can be discussed.

The same structure–soil system of Fig. 3-1 is shown, but with a rigid base in Fig. 3-5. In this case, the total motion at the base $\{u_b^t\}$ can be expressed as a function of the total rigid-body motions of a point O $\{u_o^t\}$ as

$$\{u_b^t\} = [A]\{u_o^t\} \tag{3.13}$$

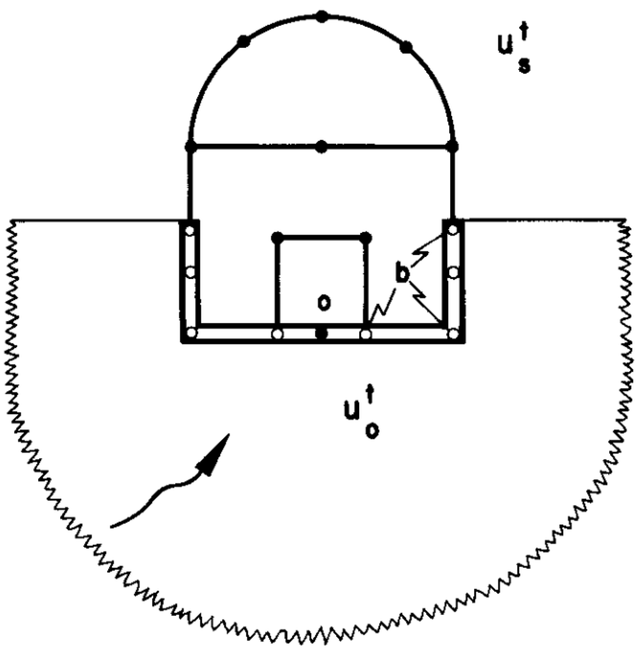
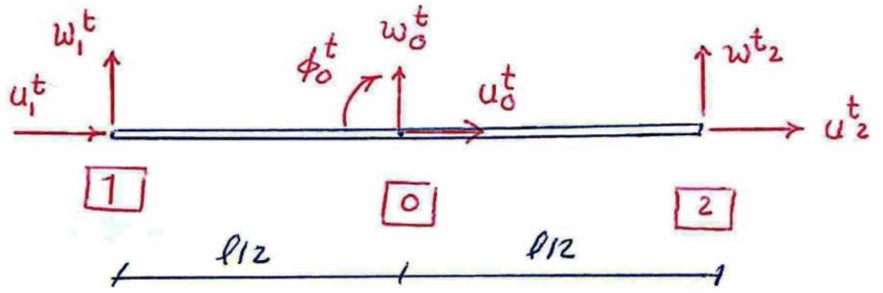


Figure 3-5 Structure–soil system with rigid base.

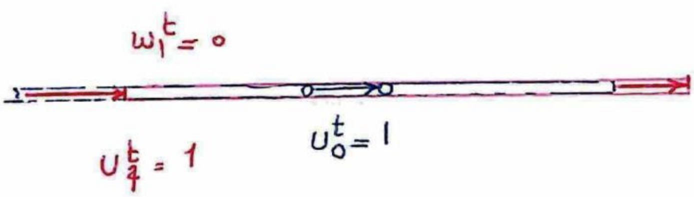
For a three-dimensional base, $\{u_o^t\}$ contains the amplitudes of three displacements and three rotations. The matrix $[A]$ represents the kinematic transformation with geometric quantities only. In Problem 3.1 the $[A]$-matrix for a two-dimensional case is presented.

$$\{u_b^t\} = \begin{Bmatrix} u_1^t \\ w_1^t \\ u_2^t \\ w_2^t \end{Bmatrix} \quad ; \quad \{u_0^t\} = \begin{Bmatrix} u_0^t \\ w_0^t \\ \phi_0^t \end{Bmatrix}$$



$\{u_0^t\} = \downarrow$

$$\begin{Bmatrix} u_0^t = 1 \\ w_0^t = 0 \\ \phi_0^t = 0 \end{Bmatrix} \text{مرحله اول}$$

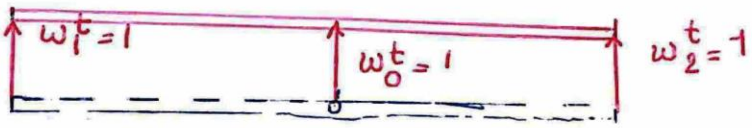


ستون اول $[A]$:

$$\{u_b^t\} = \begin{Bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{Bmatrix}$$

$\downarrow$

$$\begin{Bmatrix} u_0^t = 0 \\ w_0^t = 1 \\ \phi_0^t = 0 \end{Bmatrix} \text{مرحله دوم}$$

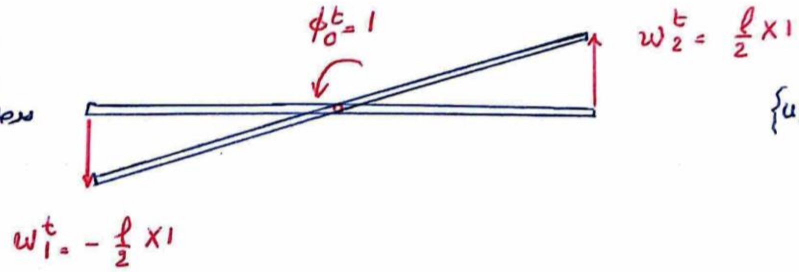


ستون دوم $[A]$

$$\{u_b^t\} = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{Bmatrix}$$

$\downarrow$

$$\begin{Bmatrix} u_0^t = 0 \\ w_0^t = 0 \\ \phi_0^t = 1 \end{Bmatrix} \text{مرحله سوم}$$



ستون سوم $[A]$

$$\{u_b^t\} = \begin{Bmatrix} 0 \\ l/2 \\ 0 \\ -l/2 \end{Bmatrix}$$

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l/2 \\ 1 & 0 & 0 \\ 0 & 1 & l/2 \end{bmatrix}$$

For a rigid base, the motion of the structure–soil interface which represents the boundary between the two subsystems thus depends only on $\{u_o^t\}$. Compared to a flexible base, the number of degrees of freedom is reduced. In the substructure of the soil with excavation (system g), the compatibility constraints of the rigid base are enforced (Fig. 3-6). The base is massless (dashed line). Obviously, the free-field motion is unchanged.

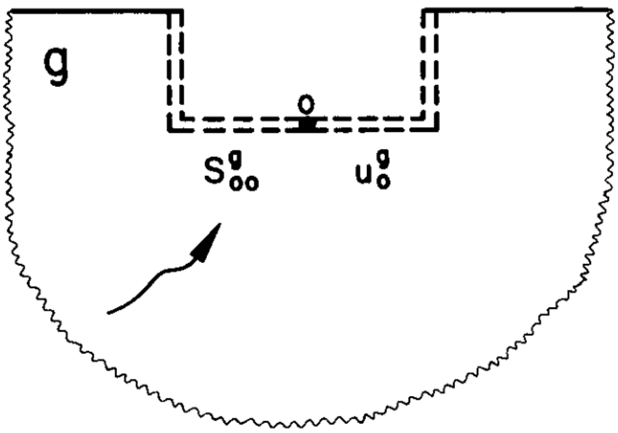


Figure 3-6 Reference soil system with excavation and rigid structure–soil interface.

Introducing this transformation of variables,

$$\begin{Bmatrix} \{u_s^t\} \\ \{u_b^t\} \end{Bmatrix} = \begin{bmatrix} [I] & \\ & [A] \end{bmatrix} \begin{Bmatrix} \{u_s^t\} \\ \{u_o^t\} \end{Bmatrix} \tag{3.14}$$

in Eq. 3.9 and premultiplying by the transposed transformation matrix defined by Eq. 3.14 leads to

$$\left[\begin{array}{c:c} [S_{ss}] & [S_{so}] \\ [S_{os}] & [S_{oo}^s] + [S_{oo}^g] \end{array}\right] \begin{Bmatrix} \{u_s^t\} \\ \{u_o^t\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [A]^T[S_{bb}^f]\{u_b^f\} \end{Bmatrix} \tag{3.15}$$

where

$$[S_{so}] = [S_{sb}][A] \tag{3.16a}$$

$$[S_{oo}^s] = [A]^T[S_{bb}^s][A] \tag{3.16b}$$

$$[S_{oo}^g] = [A]^T[S_{bb}^g][A] \tag{3.16c}$$

$[I]$ denotes the unit matrix. $[S_{oo}^s]$, $[S_{so}]$, and $[S_{os}] = [S_{so}]^T$ are the dynamic-stiffness submatrices of the structure with a rigid base. They are normally directly established when discretizing the structure by selecting models that take account of the geometric constraints of the rigid base. Equations 3.16a and b are thus not explicitly used. $[S_{oo}^g]$ represents the dynamic-stiffness matrix of the soil with excavation for a rigid structure–soil interface (Fig. 3-6). In a general three-dimensional case, $[S_{oo}^g]$ describes the amplitudes of the three forces and moments acting in point O which lead to unit amplitudes of displacements and rotations in the same point of the rigid base connected to the soil.

معادلات اندرکنش درحوزه‌ی زمان: شالوده صلب :

$[A]$: ماتریس انتقال سینماتیک

$$\{u_b^t\} = [A]\{u_0^t\}$$

$$\begin{Bmatrix}\{u_s^t\}\\ \{u_b^t\}\end{Bmatrix} = \begin{bmatrix}[I] & [0]\\ [0] & [A]\end{bmatrix}\begin{Bmatrix}\{u_s^t\}\\ \{u_0^t\}\end{Bmatrix}$$

$$\begin{bmatrix}[I] & [0]\\ [0] & [A]\end{bmatrix}^T \begin{bmatrix}[S_{ss}] & [S_{sb}]\\ [S_{bs}] & [S_{bb}^s]+[S_{bb}^g]\end{bmatrix}\begin{bmatrix}[I] & [0]\\ [0] & [A]\end{bmatrix}\begin{Bmatrix}u_s^t\\ u_0^t\end{Bmatrix} = \begin{bmatrix}[I] & [0]\\ [0] & [A]\end{bmatrix}^T\begin{Bmatrix}\{0\}\\ [S_{bb}^f][u_b^f]\end{Bmatrix}$$

$$\begin{bmatrix}[I] & [0]\\ [0] & [A]^T\end{bmatrix}\begin{bmatrix}[S_{ss}] & [S_{sb}][A]\\ [S_{bs}] & ([S_{bb}^s]+[S_{bb}^g])[A]\end{bmatrix}\begin{Bmatrix}\{u_s^t\}\\ \{u_0^t\}\end{Bmatrix} = \begin{bmatrix}[I] & [0]\\ [0] & [A]^T\end{bmatrix}\begin{Bmatrix}\{0\}\\ [S_{bb}^f]\{u_b^f\}\end{Bmatrix}$$

$$\left[\begin{array}{c|c}[S_{ss}] & [S_{sb}][A]\\ [A]^T[S_{bs}] & [A^T]([S_{bb}^s]+[S_{bb}^g])[A]\end{array}\right]\begin{Bmatrix}\{u_s^t\}\\ \{u_0^t\}\end{Bmatrix} = \begin{Bmatrix}\{0\}\\ [A]^T[S_{bb}^f]\{u_b^f\}\end{Bmatrix}$$

$$\rightarrow \begin{bmatrix}[S_{ss}] & [S_{s0}]\\ [S_{0s}] & [S_{00}^s]+[S_{00}^g]\end{bmatrix}\begin{Bmatrix}\{u_s^t\}\\ \{u_0^t\}\end{Bmatrix} = \begin{Bmatrix}\{0\}\\ [A]^T[S_{bb}^f]\{u_b^f\}\end{Bmatrix}$$

توضیح درمورد $[S_{00}^g]$:

$$\underset{n\times 1}{\{u_b^t\}} = \underset{n\times 6}{[A]}\,\underset{6\times 1}{\{u_0^t\}}$$

$$\underset{6\times 6}{[S_{00}^g]} = \underset{6\times n}{[A^T]}\,\underset{n\times n}{[S_{bb}^g]}\,\underset{n\times 6}{[A]}$$

$$[S_{s0}] = [S_{sb}][A];$$

$$[S_{0s}] = [A]^T[S_{bs}]; \quad \rightarrow [S_{0s}] = [S_{s0}]^T$$

$$[S_{00}^s] = [A]^T[S_{bb}^s][A];$$

$$[S_{00}^g] = [A]^T[S_{bb}^g][A];$$

هر سطر یا ستون $[S_{00}^g]$ شامل سه مولفه نیرویی و سه مولفه گشتاور هستند که با اعمال تغییر مکان واحد (دوران یا جابجایی واحد) در نقطه O حاصل شده‌اند.

Equation 3.15 represents the equations expressed in total motion. As in the case of a flexible basemat, the load vector depends on the free-field motion $\{u_b^f\}$ in those nodes that will subsequently lie on the structure–soil interface. The load vector $[A]^T[S_{bb}^f]\{u_b^f\}$, which in a three-dimensional case consists of three forces and three moments, acts in point O of the rigid base. No need really exists to calculate the motion of any other reference soil system.

To gain insight into the physical significance of the load vector, it is meaningful to calculate the seismic motion of the ground system (soil accounting for the excavation) with the compatibility constraints of the rigid base enforced. The seismic motion of this reference subsystem shown in Fig. 3-6 is denoted as $\{u_o^g\}$ and represents a scattered wave motion. The compatibility constraints of the rigid base for the soil–subsystem g are formulated analogously as in Eq. 3.13:

$$\{u_b^g\} = [A]\{u_o^g\} \tag{3.17}$$

Substituting this equation in Eq. 3.8, premultiplying by $[A]^T$, and using Eq. 3.16c results in

$$[S_{oo}^g]\{u_o^g\} = [A]^T[S_{bb}^f]\{u_b^f\} \tag{3.18}$$

or

$$\{u_o^g\} = [S_{oo}^g]^{-1}[A]^T[S_{bb}^f]\{u_b^f\} \tag{3.19}$$

This equation can, of course, be derived directly from Eq. 3.15, deleting all matrices associated with the structure, and setting $\{u_o^t\} = \{u_o^g\}$. As $\{u_b^f\}$ along the walls of the embedded structure varies with depth, a rotational component is also present in $\{u_o^g\}$ even for vertically propagating shear waves.

Substituting Eq. 3.18 in Eq. 3.15 leads to the equivalent of Eq. 3.5 for a structure with a rigid base.

$$\begin{bmatrix} [S_{ss}] & [S_{so}] \\ [S_{os}] & [S_{oo}^s] + [S_{oo}^g] \end{bmatrix} \begin{Bmatrix} \{u_s^t\} \\ \{u_o^t\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [S_{oo}^g]\{u_o^g\} \end{Bmatrix} \tag{3.20}$$

This equation in total motion can be physically interpreted as illustrated in Fig. 3-7. The discretized structure specified by $[S^s]$ is supported on a generalized

پیش از این نیز داشتیم:

$$[S_{bb}^f]\{u_b^f\} = [S_{bb}^g][u_b^g]$$

$$\{u_b^g\} = [A]\{u_B^g\}$$

$$\Rightarrow [A^T][S_{bb}^f]\{u_b^f\} = [A^T][S_{bb}^g][A]\{u_b^g\}$$

$$\Rightarrow \qquad = [S_{oo}^g]\{u_o^g\}$$

$$\Rightarrow \begin{bmatrix} [S_{ss}] & [S_{so}] \\ [S_{os}] & [S_{oo}^s]+[S_{oo}^g] \end{bmatrix} \begin{Bmatrix} \{u_s^t\} \\ \{u_o^t\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [S_{oo}^g]\{u_o^g\} \end{Bmatrix}$$

$$\{u_o^g\} = [S_{oo}^g]^{-1}[A^T][S_{bb}^f]\{u_b^f\}$$

☑ $[S_{ss}]$ و $[S_{os}]$ در دینامیک سازه‌ها با استفاده از آنالیز ماتریسی و وابسته کردن درجات آزادی تقریباً قابل محاسبه هستند و بنابراین در عمل از فرمول‌های فوق محاسبه می‌شوند. چون از ابتدا ماتریس سختی با توجه به درجات آزادی مستقل از هم تشکیل می‌شود

☑ $[A^T][S_{bb}^f]\{u_b^f\}$: بردار نیرو شامل سه نیرو و سه لنگر در راستا که در نقطه O اعمال می‌شوند.

$$[6\times n_b][n_b\times n_b]\{n_b\times 1\} = \{6\times 1\}$$

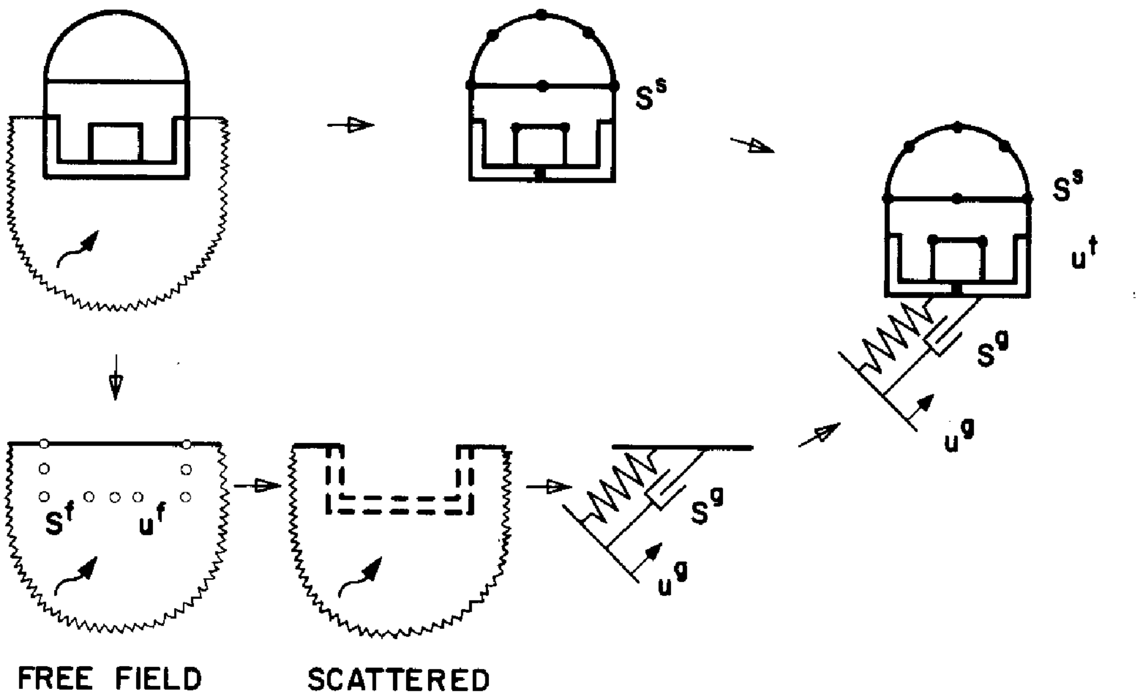


Figure 3-7 Physical interpretation of basic equation of motion in total displacements (scattered waves).

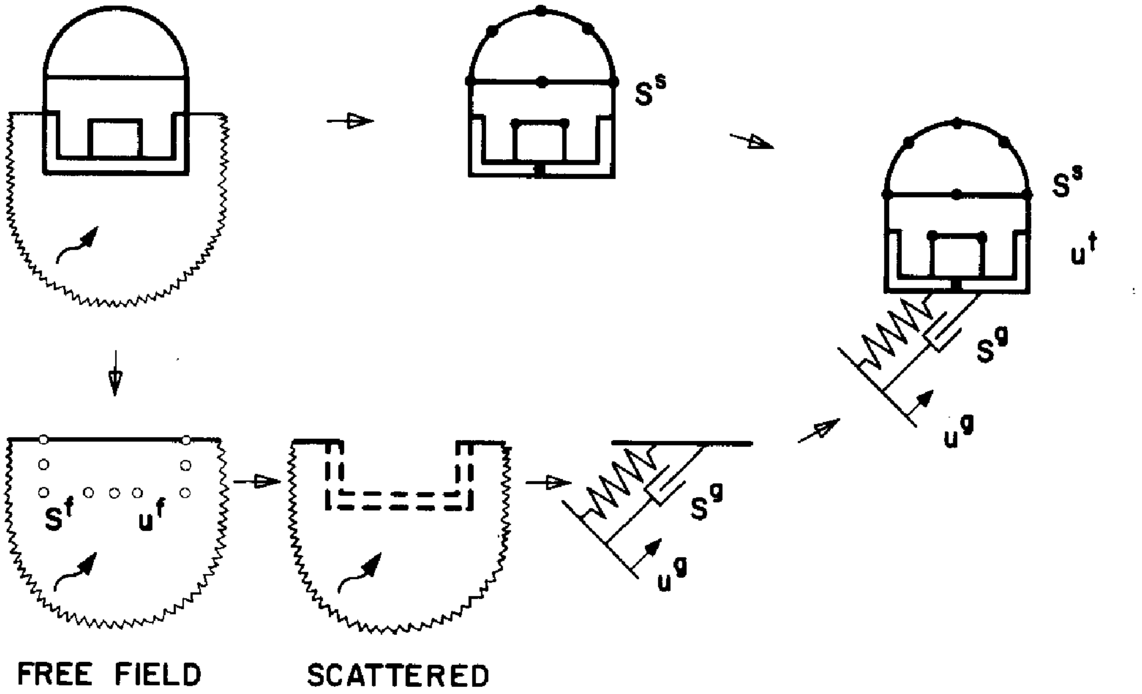


Figure 3-7 Physical interpretation of basic equation of motion in total displacements (scattered waves).

spring characterized by $[S^g_{oo}]$. The end of the generalized spring not connected to the structure is excited by $\{u^g_o\}$, which is calculated from $\{u^f_b\}$ and $[S^f_{bb}]$. This interpretation is also valid with minor adjustments for the structure with a flexible base (Eq. 3.5). The vector and matrix symbols, as well as the subscripts, have been deleted in Fig. 3-7. By setting $\{u^t_o\} = \{0\}$ in Eq. 3-20, it can be deduced that $[S^g_{oo}]\{u^g_o\}$ represents the amplitudes of the forces exerted on the rigid base in point *O* by the seismic motion when the base is kept fixed. They are sometimes called driving forces. Equation 3.5 can be interpreted analogously for a structure with a flexible base.